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INFLUENCE OF DIFFUSELY SPECULAR REFLECTION ON THE TRANSFER PROCESS IN A GAP

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UDC 533.6.011.8

A quasidiffusion approximation is constructed for the problem of radiation transfer in a narrow gap in the case of diffusely specular reflection by the walls. The accuracy of the approximation obtained is investigated.

The quasidiffusion approximation proposed by Smoluchowski [1] is used extensively in studying the radiant or free-molecule transfer in long channels. It is shown in [2, 3] that this method can be extended to the transfer problem in the narrow gap between parallel plates. The assumption about the diffuse nature of the radiation played a substantial part in these papers. However, there is a significant quantity of experimental data indicating that the reflexivity of many materials has a substantial specular component: $\rho = \rho^s + \rho^d$. As is shown in [4], the problem of determining the effective fluxes reduces in this case to the numerical solution of an integral equation. An assumption about the smallness of the gap is made in this paper that affords a possibility of constructing a quasidiffusion approximation that allows of analytical solution in a number of cases.

1. PLANE-PARALLEL GAP

Let us consider a domain V, the gap between two plane-parallel walls, each of which occupies a domain S bounded by a contour L in a plane. A diffuse flux of density Q, homogeneous along the height of the gap and dependent only on the location of the point on the con-

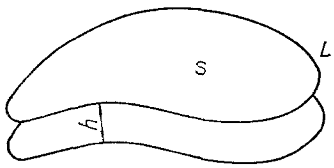

Fig. 1. Diagram of the domain under consideration.

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 4, pp. 675-681, April, 1985. Original article submitted January 3, 1984.

 0022-0841/85/4804-0504$09.50

tour L is incident on the side surface of the gap from outside (Fig. 1). Part of the flux reflected from the i-th wall at the point y is diffuse, and will be denoted by $q^d_i(y)$ ($i = 1, 2$, $y \in S_i$). For simplicity we assume no absorption on the walls, i.e., $\rho = 1$, then $\rho^s_i + \rho^d_i = 1$.

The function $\hat{q}_i = q^d_i + q^*_i$, where q^*_i is the intrinsic radiation flux density of the i-th wall. The method described in [4] results in a system of two linear integral equations in $\hat{q}_i(y)$ ($i = 1, 2$) for the domain under consideration:

$$\hat{q}_i(\mathrm{y}) = q_i^*(\mathrm{y}) + \rho^d \left(\sum_{j=1}^{2} \int_S \hat{q}_j(\mathrm{y}') K_{ij}(\mathrm{y}, \mathrm{y}') dS_{\mathrm{y}'} + B_i(\mathrm{y}) \right), \tag{1}$$

where

$$K_{ij}(\mathrm{y}, \mathrm{y}') = \frac{h^2}{\pi} \sum_{k=0}^{\infty} (\rho_1^s \rho_2^s)^k M_{ij}(\mathrm{y}, \mathrm{y}');$$

$$M_{ij}(\mathrm{y}, \mathrm{y}') = \begin{Vmatrix} \dfrac{\rho_2^s (2k+2)^2}{[h^2(2k+2)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2} & \dfrac{(2k+1)^2}{[h^2(2k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2} \\ \dfrac{(2k+1)^2}{[h^2(2k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2} & \dfrac{\rho_1^s (2k+2)^2}{[h^2(2k+2)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2} \end{Vmatrix};$$

$$B_i(\mathrm{y}) = \frac{1}{2\pi} \int_L Q(\mathrm{y}')(\mathrm{n}_{\mathrm{y}'}, \mathrm{y} - \mathrm{y}') \Bigg[\frac{1}{|\mathrm{y} - \mathrm{y}'|^2} - \sum_{k=0}^{\infty} (\rho_1^s \rho_2^s)^k \left(\frac{\rho_{3-i}^d}{h^2(2k+1)^2 + (\mathrm{y} - \mathrm{y}')^2} + \frac{\rho_{3-i}^d \rho_i^d}{h^2(2k+2)^2 + |\mathrm{y} - \mathrm{y}'|^2} \right) \Bigg] dL_{\mathrm{y}'}.$$

In the particular case $\rho^d_1 = \rho^d_2 = \rho^d$, the system of integral equations decomposed into two independent equations after the insertion of the substitutions $u = \hat{q}_1 + \hat{q}_2$, $v = -\hat{q}_1 + \hat{q}_2$, $u^* = q^*_1 + q^*_2$, $v^* = -q^*_1 + q^*_2$:

$$u = u^* + \rho^d \left[\int_S u(\mathrm{y}') K_u(\mathrm{y}, \mathrm{y}') dS_{\mathrm{y}'} + \int_L Q(\mathrm{y}') K_0(\mathrm{y}, \mathrm{y}')\, dL_{\mathrm{y}'} \right], \tag{2}$$

$$v = v^* - \rho^d \int_S v(\mathrm{y}') K_v(\mathrm{y}, \mathrm{y}') dS_{\mathrm{y}'}; \tag{3}$$

$$K_u = \frac{h^2}{\pi} \sum_{k=0}^{\infty} \frac{(\rho^s)^k (k+1)^2}{[h^2(k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2}; \quad K_v = \frac{h^2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho^s)^k (k+1)^2}{[h^2(k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2};$$

$$K_0 = \frac{(\mathrm{n}_{\mathrm{y}'}, \mathrm{y} - \mathrm{y}')}{\pi} \left(\frac{1}{|\mathrm{y} - \mathrm{y}'|^2} - \rho^d \sum_{k=0}^{\infty} \frac{(\rho^s)^k}{[h^2(k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2]^2} \right).$$

Let us define a two-dimensional component of the spherical radiation vector in the plane of the gap, a vector **E** averaged over the height of the gap:

$$\mathrm{E}(\mathrm{y}) = \frac{1}{2\pi h} \int_S u(\mathrm{y}')(\mathrm{y} - \mathrm{y}') \sum_{k=0}^{\infty} (\rho^s)^k \left[\frac{1}{h^2 k^2 + |\mathrm{y} - \mathrm{y}'|^2} - \frac{1}{h^2(k+1)^2 + |\mathrm{y} - \mathrm{y}'|^2} \right] dS_{\mathrm{y}'} + \frac{1}{\pi} \int_L \frac{Q(\mathrm{y}')(\mathrm{n}_{\mathrm{y}'}, \mathrm{y} - \mathrm{y}')(\mathrm{y} - \mathrm{y}')}{|\mathrm{y} - \mathrm{y}'|^3} \Bigg[\frac{\pi}{2} - \operatorname{arctg} \frac{|\mathrm{y} - \mathrm{y}'|}{h} - \sum_{k=1}^{\infty} \left((k+1) \operatorname{arctg} \frac{|\mathrm{y} - \mathrm{y}'|}{(k+1)h} - 2k \operatorname{arctg} \frac{|\mathrm{y} - \mathrm{y}'|}{kh} + (k-1) \operatorname{arctg} \frac{|\mathrm{y} - \mathrm{y}'|}{(k-1)h} \right) \Bigg] dL_{\mathrm{y}'}. \tag{4}$$

2. QUASIDIFFUSION APPROXIMATION

Equations (2) and (3) can be solved numerically. In the case of a small gap, i.e., when $h \ll l$, the solution is made considerably more difficult by the δ-like nature of the kernel K_u. However, precisely the local nature of the kernel indeed permits replacement of the problem by a simpler problem that allows analytic solution in a number of cases.

Let h << l. The integral equation can be replaced by differential equations by using Taylor series expansions of the unknown function. By analogy with the case of diffuse reflection, we use the notation

$$P^{(1)}(\mathbf{y}) = \rho^d \sum_{k=0}^{\infty} (\rho^s)^k (k+1)^2 \Pi^{(1)}_{k+1}(\mathbf{y}), \tag{5}$$

where

$$\Pi_k^{(1)}(\mathbf{y}) = \begin{Vmatrix} \frac{1}{2\pi}\int_0^{2\pi} \cos^2\psi f_1(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_1(\psi)\,d\psi \\ \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_1(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin^2\psi f_1(\psi)\,d\psi \end{Vmatrix};$$

$$f_1(\psi) = \frac{1}{2}\left[\ln\left(1+\frac{a^2}{k^2h^2}\right) - \frac{a^2}{k^2h^2+a^2}\right].$$

The constant that is the approximate solution of (3), $v^0 = v^*(2-\rho^d)/2$ can be indicated and which upon substitution in (3) yields a residual on the order of $O(h^2/l^2)$.

Here (2) is converted as follows:

$$h^2 \operatorname{div}(P^{(1)} \operatorname{grad} u) = -u^*. \tag{6}$$

The expression (6) can also be obtained from the radiant energy balance equation

$$h \operatorname{div} \mathbf{E} = u^*. \tag{7}$$

If u in (4) is expanded in a Taylor series in the neighborhood of y, then to the accuracy of the infinitesimals $O(h^2/l^2)$ far from the boundary

$$\mathbf{E} = -hP^{(2)} \operatorname{grad} u, \tag{8}$$

where

$$P^{(2)}(\mathbf{y}) = \rho^d \sum_{k=0}^{\infty} (\rho^s)^k (k+1)^2 \Pi^{(2)}_{k+1}(\mathbf{y});$$

$$\Pi_k^{(2)}(\mathbf{y}) = \begin{Vmatrix} \frac{1}{2\pi}\int_0^{2\pi} \cos^2\psi f_2(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_2(\psi)\,d\psi \\ \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_2(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin^2\psi f_2(\psi)\,d\psi \end{Vmatrix};$$

$$f_2(\psi) = \frac{1}{2}\ln\left(1+\frac{a^2}{k^2h^2}\right).$$

Let us note that the differences between corresponding components of the tensors $h^2P^{(1)}$ and $h^2P^{(2)}$ are on the order of $O(h^2/l^2)$. The accuracy of results obtained for different representations of $P^{(\nu)}$ (ν = 1, 2) will be compared below.

Let us obtain the boundary condition for extraction of the necessary solution of (6). It can be deduced from the integral equation (2) if the function is is expanded in a Taylor series in the neighborhood of a point lying on the contour L:

$$\frac{h}{\rho^d}\frac{\partial u}{\partial n} = (-u(\mathbf{y}_0) + 2\rho^d Q(\mathbf{y}_0))(1+\varkappa(\mathbf{y}_0)) + 2u^*, \tag{9}$$

where

$$\varkappa = \begin{cases} \dfrac{h}{2R(\mathbf{y}_0)}, & \text{if } \mathbf{y}_0 \text{ is a convexity point of the contour L,} \\ 0, & \text{if } \mathbf{y}_0 \text{ is a concavity point of the contour L.} \end{cases}$$

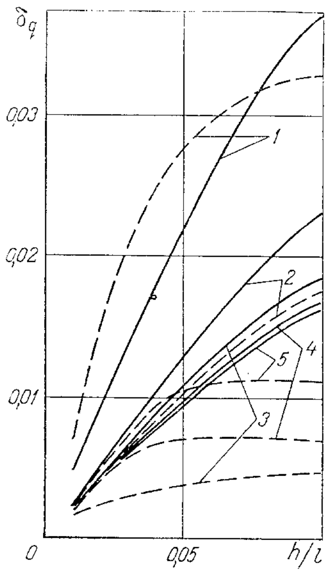

Fig. 2. Dependence of the relative error in determining the effective flux on the dimensionless magnitude of the gap for different values of ρ^d: 1) $\rho^d = 0.25$; 2) 0.5; 3) 0.75; 4) 0.9; 5) 1. The dashed and solid curves correspond to the transfer coefficients $P^{(1)}$ and $P^{(2)}$.

Therefore, the problem of a quasidiffusion approximation for the case of diffusely specular reflection reduces to solving the Poisson equation (6) in the domain S with the boundary condition (9) on the contour L. Knowing u and setting $v \approx v^0$, we can easily determine the effective fluxes on the walls as well as the energy flux E within the domain S by means of formula (8). The expression for E_n at points on the contour L can be obtained from (4) by using the expansion of the unknown function u in a Taylor series

$$E_n(\mathbf{y}_0) = \frac{1}{2\rho^d} u(\mathbf{y}_0) - hP_{ik}^{(\nu)}(\mathbf{y}_0) \frac{\partial u}{\partial x_k} n_i - Q(\mathbf{y}_0), \tag{10}$$

where $P^{(\nu)}{}_{ik}$ are components of the tensor $P^{(\nu)}$ ($\nu = 1, 2$), n_i are components of the vector **n**, and summation from 1 to 2 is understood to be over the repeated subscripts i, k.

3. PARTICULAR CASE: GAP BETWEEN PARALLEL STRIPS

Let domain V be the gap between infinite plane strips each of which is bounded by lines $x = 0$ and $x = l$ in its plane. Fluxes of density Q_1 at $x = 0$ and Q_2 at $x = l$ are incident on the side surface from outside.

For an infinite strip the differential equations (6) go over into

$$h^2 \frac{d}{dx}\left[P^{(\nu)}(x)\frac{du}{dx}\right] = -u^* \tag{11}$$

with the boundary conditions

$$\begin{aligned} \frac{h}{\rho^d}\frac{\partial u(0)}{\partial x} &= -u(0) + 2\rho^d Q_1 + 2u^*, \\ -\frac{h}{\rho^d}\frac{\partial u(l)}{\partial x} &= -u(l) + 2\rho^d Q_2 + 2u^*. \end{aligned} \tag{12}$$

The solution has the form

$$u(x) = -\frac{u^*}{h^2}\int_0^x \frac{x'dx'}{P^{(\nu)}(x')} + \frac{c}{h^2}\int_0^x \frac{dx'}{P^{(\nu)}(x')} + b, \tag{13}$$

where

$$c = \frac{u^*\left(\frac{1}{h}\int_0^l \frac{x'dx'}{P^{(\nu)}(x')} + \frac{l}{\rho^d P^{(\nu)}(l)}\right) + 2\rho^d h(Q_2 - Q_1)}{\frac{1}{h}\int_0^l \frac{dx'}{P^{(\nu)}(x')} + \frac{1}{\rho^d}\left(\frac{1}{P^{(\nu)}(0)} + \frac{1}{P^{(\nu)}(l)}\right)};$$

$$b = \frac{c}{\rho^d hP^{(\nu)}(0)} + 2(\rho^d Q_1 + u^*).$$

By knowing u and setting $v \approx v^0$, we can easily determine the flux $\hat{q}_i$. Let us note that the effective flux consists of $\hat{q}_i$ and q^s parts of the flux being reflected specularly from the i-th wall. Since

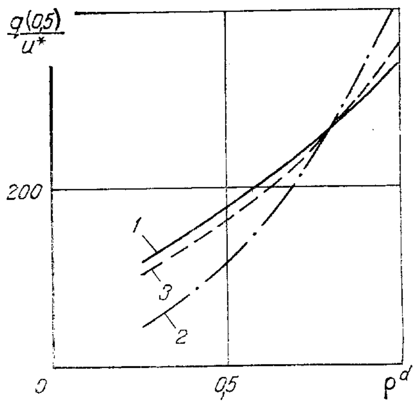

Fig. 3

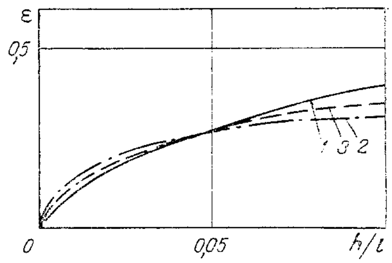

Fig. 4

Fig. 3. Dependence of the dimensionless effective flux on ρ^d for $u^* \neq 0$: 1) numerical solution of the integral equation; 2, 3) analytic solutions of the differential equation with transfer coefficients $P^{(1)}$ and $P^{(2)}$, respectively.

Fig. 4. Dependence of the emissivity of the side surface of the gap (a dimensionless quantity) on the dimensionless magnitude of the gap. Same notation as in Fig. 3.

$$q^s = \frac{\rho^s}{\rho^d}\, q^d = \frac{\rho^s}{\rho^d}(\hat{q} - q^*);\quad q = \hat{q} + \frac{\rho^s}{\rho^d}(\hat{q} - q^*) = \frac{\hat{q}}{\rho^d} + q^*\left(1 - \frac{1}{\rho^d}\right).$$

Taking into account the definition of the flux u, the effective fluxes on the walls can be expressed in terms of u and q^*_i, as follows:

$$q_1 = \frac{u}{2\rho^d} + \frac{q_1^*}{4}\left(3 - \frac{2}{\rho^d}\right) - \frac{q_2^*}{4}\left(\frac{2}{\rho^d} - 1\right),$$

$$q_2 = \frac{u}{2\rho^d} - \frac{q_1^*}{4}\left(\frac{2}{\rho^d} - 1\right) + \frac{q_2^*}{4}\left(3 - \frac{2}{\rho^d}\right). \tag{14}$$

For an infinite strip we introduce the quantity ε

$$\varepsilon = \frac{E_x(0)}{Q_1} \quad \text{or} \quad u^* = 0,\ Q_2 = 0. \tag{15}$$

4. COMPARISON WITH THE RESULTS OF A NUMERICAL SOLUTION OF THE INTEGRAL EQUATION

For the domain examined in Sec. 3, a numerical solution of the integral equation (2) was obtained by the Krylov–Bogolyubov method [5] under the assumption of diffusely specular reflection. The error in the solution was estimated by the Runge method and does not exceed 0.5%. Therefore, the possibility exists for comparing the accuracy with which the analytic solutions of the problem of a quasidiffusion approximation (6) and (9) approximate the solution of the integral equation (2) for two different representations of the transfer coefficient $P^{(\nu)}$ ($\nu = 1, 2$).

For $u^* = 0$ the values of the function $q(x/l)/Q_1$ are practically in agreement with the analytical solutions ($\nu = 1, 2$) in the case of the numerical solution of the integral equation.

Of special interest is a study of the behavior of the error in the analytic solution as a function of ρ^d in the absence of intrinsic radiation ($u^* = 0$). As seen from Fig. 2, for $\nu = 2$ the relative error in determining the effective flux decreases monotonically with the growth of ρ^d, and is 1.5% for $\rho^d = 1$. In the case when $\nu = 1$, the dependence is more complex in nature. A decrease in the error is observed as ρ^d increases to 0.75. As ρ^d grows further, the error starts to grow also, however, without exceeding 1%.

For $u^* \neq 0$, the greatest error is observed at the point $x/l = 0.5$. It is shown in Fig. 3 that $P^{(2)}$ yields a considerably better approximation of the ratio $q(0.5)/u^*$ for practically all values of ρ^d for $u^* \neq 0$.

For the emissivity ε of the gap side surface (see Fig. 4), the representation of the permeability in the form $P^{(2)}$ also turns out to be more exact.

The results presented permit making the deduction that the analytic solution (6) and (9) obtained yield an approximate solution of the transfer problem in a narrow gap with acceptable accuracy for practice.

NOTATION

$a(\psi)$, distance between the point y and contour L in a direction making the angle $\pi + \psi$ with the ox axis; b, c, arbitrary constants; h, distance between the walls of the gap; E, two-dimensional component of the spherical radiation vector in the plane of the gap; L, contour of domain S; l, width of an infinite strip; **n**, unit normal vector to contour L; Q, Q_1, Q_2, radiation flux densities incident from outside; q_1, q_2, effective flux densities on the walls; $\hat{q}_i = q^d{}_i + q^*{}_i$; $P^{(\nu)}$, transfer coefficient tensor; $R(y_0)$, radius of curvature of contour L at point y_0; S, two-dimensional domain; $u = \hat{q}_1 + \hat{q}_2$, $v = -\hat{q}_1 + \hat{q}_2$; V, gap domain; y, point of domain S; ε, emissivity of the gap side surface; ρ, reflectivity. The super and subscripts are: d, diffuse component; s, specular component; and ν, i, representation numbers for the transfer coefficient and the gap walls.

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